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**COVER** Accelerating optical beams have attracted a great deal of interest in the past several years, fueling the research in beam synthesis as well as other areas beyond optics. An intriguing question arises naturally: can we design accelerating beams that propagate along arbitrary trajectories and yet have controllable symmetric transverse profiles? In this issue, Chen's group provides a comprehensive review of recent work on various spatially-shaped accelerating beams along pre-designed trajectories, including self-accelerating, self-breathing, and self-propelling Bessel-like beams. They also discuss the potential application of these dynamical beams in optical trapping and manipulation. The cover figure illustrates an example of a specially-designed accelerating beam curving and propelling along a three-dimensional trajectory in free space (see the review by Juanying Zhao et al. on page 1157).



Volume 60 Number 13  
July 2015

## EDITORIAL

- 1139 **Editor's note: how and where does continental crust form?**  
Yaoling Niu

## REVIEWS

### Earth Sciences

- 1141 **Continental crust formation at arcs, the arclogite “delamination” cycle, and one origin for fertile melting anomalies in the mantle**  
Cin-Ty A. Lee • Don L. Anderson

### Physics & Astronomy

- 1157 **Specially shaped Bessel-like self-accelerating beams along pre-designed trajectories**  
Juanying Zhao • I. D. Chremmos • Ze Zhang • Yi Hu • Daohong Song • Peng Zhang • N. K. Efremidis • Zhigang Chen

### Materials Science

- 1170 **Construction of smart inorganic nanoparticle-based ultrasound contrast agents and their biomedical applications**  
Ming Ma • Hangrong Chen • Jianlin Shi



## ARTICLES

### Life & Medical Sciences

- 1184 **Phylogeography of *Haplocarpha rueppelii* (Asteraceae) suggests a potential geographic barrier for plant dispersal and gene flow in East Africa**

Ling-Yun Chen • John K. Muchuku • Xue Yan • Guang-Wan Hu • Qing-Feng Wang

p1193

### Chemistry

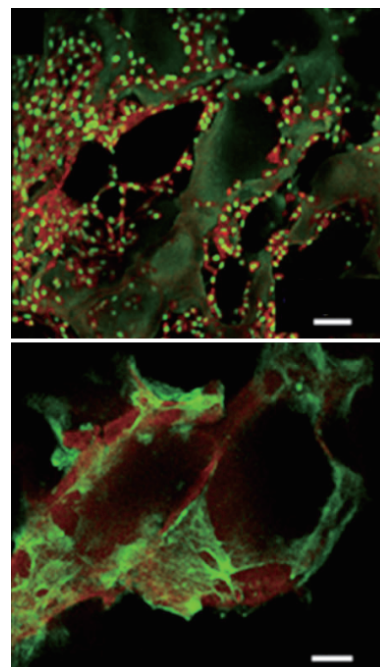
- 1193 **Cytocompatible 3D chitosan/hydroxyapatite composites endowed with antibacterial properties: toward a self-sterilized bone tissue engineering scaffold**

Yongjun Qiao • Zhongjun Zhai • Limei Chen • Hong Liu

### Engineering Sciences

- 1203 **Self-powered macroscopic Brownian motion of spontaneously running liquid metal motors**

Bin Yuan • Sicong Tan • Yixin Zhou • Jing Liu



## LETTERS

### Physics & Astronomy

- 1211 **Laser-driven plasma collider for nuclear studies**

Changbo Fu • Jie Bao • Liming Chen • Jianjun He • Long Hou • Liang Li • Yanfei Li • Yutong Li • Guoqian Liao • Yongjoo Rhee • Yang Sun • Shiwei Xu • Dawei Yuan • Xiaopeng Zhang • Gang Zhao • Jiarui Zhao • Baojun Zhu • Jianqiang Zhu • Jie Zhang

### Materials Science

- 1214 **Magnetic anisotropy of Fe films deposited by dc magnetron sputtering under an external magnetic field**

Jiahui Chen • Jing Ma • Liang Wu • Yang Shen • Ce-Wen Nan

## RESEARCH HIGHLIGHTS

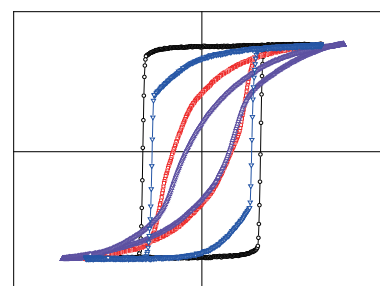
- 1218 **Sensing system for mimicking cancer cell–drug interaction**

Yuliang Zhao

- 1220 **The mechanism behind the DOM effects on methylmercury photodegradation**

Khan M. G. Mostofa • Cong-Qiang Liu • Marco Minella • Davide Vione

p1214



## NEWS & VIEWS

- 1222 **Changes in East Asian summer monsoon and summer rainfall over eastern China during recent decades**

Renhe Zhang

## 大陆地壳究竟在哪形成, 如何形成?

牛耀龄

编者按 p1139

## 大陆弧陆壳的形成, 陆弧榴辉岩的拆沉循环及异常富集地幔熔体的成因

Cin-Ty A. Lee • Don L. Anderson

以美国加州内华达山脉复合岩基(Sierra Nevada Batholith)为例, 证明成熟陆壳的化学成分可由地幔岩浆分异-石榴辉石岩质堆晶形成-拆沉-岩浆补给-岩浆分异这一循环过程来解释, 并同时建立了一个简单的物理模型用以阐释拆沉作用. 在这个模型里, 陆弧的岩浆作用持续进行, 伴随岩浆分异以及地壳底部石榴辉石岩质堆晶层的生长, 每隔10~30 My, 这一过程被拆沉作用打断, 石榴辉石岩质堆晶层拆沉进入地幔并伴随岩浆补给. 我们估计被拆沉进入地幔的陆弧石榴辉石岩通量大约是俯冲洋壳通量的5%~20%. 被拆沉的石榴辉石岩具有可衰变到地幔储区的同位素特征所需要的微量元素组成, 因此陆弧石榴辉石岩也可以是形成地幔储区的贡献之一. 此外, 由于辉石岩的起始熔融温度较低, 这种岩石通常最先被熔融, 并可能成为一些热点(hot spot)岩浆的主要组成.

评述 p1141

## 自加速类贝塞尔光束: 可沿着预设空间轨道传输的光束

赵娟莹, I. D. Chremmos, 张泽, 胡毅, 宋道红, 张鹏, N. K. Efremidis, 陈志刚

综述了多种特殊设计的自加速类贝塞尔光束. 这些光束在理论上通过相位调制或叠加等方法产生, 并经过实验装置得以验证. 其范围覆盖傍轴和大角度自弯曲非傍轴情况, 具体包括类贝塞尔光束、自呼吸型类贝塞尔光束、涡旋型类贝塞尔光束、自螺旋型类贝塞尔光束, 以及非傍轴类贝塞尔光束. 基于这些光束具有无衍射、自加速、自修复、光场中心对称及光束传输轨迹可控等特点, 它们不仅具有重要的基础研究价值, 而且在微粒操控、等离子体、大气科学、生物操控等诸多领域都具有重要的应用前景.

评述 p1157

## 智能无机纳米超声造影剂的构建和诊疗应用

马明, 陈航榕, 施剑林

超声造影成像因其非侵入性、风险低、价格低和轻便快捷等优势, 在肿瘤诊断方面得到了广泛的应用. 伴随着分子影像技术的进步和发展, 针对肿瘤的靶向超声造影剂的制备和应用成为材料和医学界的研究热点. 然而, 由于粒径较大和结构稳定性差等因素, 目前常用的微泡造影剂在体内循环和成像时间较短, 同时难以渗透到肿瘤组织和细胞内部实现有效的肿瘤造影成像. 针对以上问题, 目前国际科学界开展了氧化硅、金和氧化铁等无机基质的纳米超声造影剂的制备和应用研究, 力求在结构和成像性能等方面大幅度提高材料的超声造影性能. 本综述主要从3个方面介绍无机纳米超声造影剂的研究进展: (1) 新型实心、空心和多壁结构无机SiO<sub>2</sub>纳米超声造影剂的制备、成像机理和诊断应用研究; (2) 新型相转变智能超声造影剂的合成和诊疗应用进展; (3) 与光声造影成像、磁性能和肿瘤靶向性能复合的几种代表性多功能超声造影剂的介绍, 并着重总结目前国际上关于光热治疗和超声造影复合纳米材料的研究进展. 此外, 为了进一步提高无机超声造影剂的安全性和有效性, 提出了材料设计和临床前研究的指导性思路.

评述 p1170

## 菊科(*Haplocarpha rueppelii*)的谱系地理学研究揭示东非地区可能存在阻碍植物扩散与基因交流的地理隔离

陈凌云, John K. Muchuku, 严雪, 胡光万, 王青锋

东非地区是全球生物多样性热点之一. 菊科(*Haplocarpha rueppelii* (Sch.Bip.) Beauverd)主要分布在东非高山草甸. 本文从东非埃爾貢山、阿伯德爾山、肯尼亞山、乞力馬扎羅山、貝爾山收集了該物種有毛和無毛兩種分化類型, 共65個個體進行了DNA測序, 以檢驗造成分化的原因(內部生殖隔離和地理隔離). 結果表明兩種分化類型並不存在內部生殖隔離, 該物種至少在更新世時期已經分布于相互毗鄰的阿伯德爾山和肯尼亞山. 然而, 兩山間的基因交流卻很低, 表明它們之間可能存在阻礙植物擴散與基因交流的隔離屏障. 希望這項研究能夠激起更多學者對東非生物地理格局和生物多樣性的興趣.

論文 p1184

## 自发运动型液态金属马达的宏观自驱动布朗运动

袁彬, 谭思聪, 周一欣, 刘静

揭示了液态金属马达在碱性水溶液中类布朗运动的机制: 固-液界面接触产氢. 实验将微量铝箔(质量分数1%)融入GaIn10中, 以注射方式产生大量自主运动型微小马达. 采用高速摄像仪记录, 并基于图像处理量化进行分析. 结果表明液态金属马达呈现高速(约4 cm/s)无序的运动模式, 我们将其命名为宏观布朗运动. 不同于经典布朗现象, 宏观布朗运动系由液态金属合金产氢反应、液态金属马达间及其与溶液和基底的多重相互作用所致. 此外, 通过搭建类似于威尔逊云室的光学平台可以清晰显示液态金属马达产生的氢气轨迹, 并证实驱动马达的主要因素来自氢气气泡, 这与大尺寸液态金属机器主要受表面张力驱动的机制不同.

论文 p1203

## 用于核物理研究的激光等离子体对撞机

符长波, 鲍杰, 陈黎明, 何建军, 侯龙, 李亮, 李彦霏, 李玉同, 廖国前, Yongjoo Rhee, 孙扬, 许世伟, 袁大伟, 张杰, 张笑鹏, 赵家瑞, 朱保君, 赵刚, 朱健强

利用不同条件的强激光等离子体模拟天体环境引发了一门新的交叉学科——实验室天体物理学——的诞生. 元素的起源和演化是天体物理重要研究内容之一, 它依赖于准确的核反应截面输入. 然而在传统研究中低能核反应截面数据都是在常温常压环境中测得, 其原子核和核外电子是以束缚态存在, 与天体等离子体环境相去甚远. 核外电子云带来的电子屏蔽效应对核反应截面测量会造成很大的影响. 本文首次提出利用强激光产生的等离子体喷流对撞的方法研究低能核反应, 并在“神光”上成功地观测到氘-氘对撞产生的中子. 这种新型的“等离子体对撞机”为强激光核物理开辟了一条新的研究途径.

快讯 p1211

## 直流磁控溅射中外加磁场诱导铁薄膜产生的磁各向异性

陈家慧, 马静, 伍亮, 沈洋, 南策文

由于铁薄膜在磁隧道结、自旋阀、多铁性材料等领域中具有广泛应用, 制备取向、磁性、尤其是磁各向异性可控的铁薄膜至关重要. 本文采用在溅射过程中施加平行于基片的直流生长磁场的简便方法, 在较低温度下诱导产生了明显的磁各向异性. 采用直流磁控溅射方法分别在MgO(001)和MgO(011)基片上生长铁薄膜, 对比了在不同取向基片、是否施加生长磁场等条件下制备得到的铁薄膜的结构和磁性能. 研究发现施加生长磁场后薄膜的结晶度较好, 在MgO(001)上制备的薄膜具有四重磁各向异性, 在MgO(011)上制备出的薄膜具有单轴各向异性. 而不加生长磁场时, 两种基片上制备的薄膜均为磁各向同性. 这种方法也可以用于其他磁性薄膜的制备.

快讯 p1214

## 近几十年来东亚夏季风和中国东部夏季降水的变化

张人禾

新闻与观点 p1222



# Specially shaped Bessel-like self-accelerating beams along predesigned trajectories

Juanying Zhao · I. D. Chremmos · Ze Zhang ·  
Yi Hu · Daohong Song · Peng Zhang ·  
N. K. Efremidis · Zhigang Chen

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**Abstract** Over the past several years, spatially shaped self-accelerating beams along different trajectories have been studied extensively. Due to their useful properties such as resistance to diffraction, self-healing, and self-bending even in free space, these beams have attracted great attention with many proposed applications. Interestingly, some of these beams could be designed with controllable spatial profiles and thus propagate along various desired trajectories such as parabolic, snake-like, hyperbolic, hyperbolic secant, three-dimensional spiraling, and even self-propelling trajectories. Experimentally, such

beams are realized typically by using a spatial light modulator so as to imprint a desired phase distribution on a Gaussian-like input wave front propagating under paraxial or nonparaxial conditions. In this paper, we provide a brief overview of our recent work on specially shaped self-accelerating beams, including Bessel-like, breathing Bessel-like, and vortex Bessel-like beams. In addition, we propose and demonstrate a new type of dynamical Bessel-like beams that can exhibit not only self-accelerating but also self-propelling during propagation. Both theoretical and experimental results are presented along with a brief discussion of potential applications.

---

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## 1 Introduction

In 1979, Berry and Balazs [1] theoretically predicted a self-accelerating wave solution for the free space Schrödinger equation in the context of quantum mechanics. Such an interesting wave packet, described mathematically by an Airy function, evolves in time without spreading while accelerating transversely along a parabolic trajectory. The acceleration (or self-bending) occurs in spite of the fact that the center of gravity of these truncated waves remains invariant in agreement with Ehrenfest's theorem. This accelerating behavior can persist over long distances until diffraction effects eventually take over and can be explained through the principle of equivalence [2], in which a stationary Airy wave packet associated with a quantum mechanical particle in a constant gravitational field can be perceived as accelerating



upwards by a free-falling observer. Unfortunately, this ideal Airy wave packet in quantum physics is supposed to carry infinite energy which makes it more like a theoretical elegance rather than a physically realizable entity. The interest in this field was revived in 2007, when Christodoulides and co-workers introduced the concept of Airy wave packets into optics by theoretically proposing and experimentally demonstrating the finite-energy self-accelerating optical Airy beams [3, 4]. Since then, the interest in such nonconventional self-accelerating beams has blossomed [5–7], gifted with the ability to resist diffraction while undergoing self-acceleration and self-healing, alongside with numerous proposed applications [7–16], including particle manipulation, curved plasma generation, bending surface plasmons and electrons, single molecule imaging, and light-sheet microscopy.

In the past several years, great efforts have been made to uncover new accelerating wave solutions. In particular, apart from the paraxial Airy beams [3–5], nonparaxial self-accelerating beams in general capable of following curved trajectories with large bending angles were also found directly for the Maxwell equations and demonstrated experimentally [17–20], followed by other types of nonparaxial accelerating beams such as Mathieu and Weber beams [21–23]. Unfortunately, most of these solutions cannot be used to design beams with arbitrary trajectories. The latter are most efficiently designed using ray optics and the concept of caustics [24]. It should be noted that accelerating beams based on ray caustics are usually characterized by highly asymmetric transverse intensity profiles (such as the Airy beams with one or two oscillating tails). An intriguing question arises naturally: can we design accelerating beams that propagate along arbitrary trajectories and yet have controllable and possibly symmetric transverse profiles (such as Bessel-like or donut-shaped beam profiles)?

Earlier works have showed that Bessel-like beam patterns can be delivered along sinusoidal [25] or spiraling trajectories [26]. An even earlier work suggested snaking beams made of the series cascade of the so-called sword beams [27]. These beams are formed by a different ray structure, named conical-interference ray structure, which sets them clearly apart from the optical caustic beams. Quite recently, we proposed and demonstrated the self-accelerating Bessel-like beams with arbitrary trajectories [28, 29]. Using the concept of conical superposition, angular momentum can also be loaded onto such beams resulting in accelerating vortex Bessel-like beams [30, 31]. Indeed, tremendous efforts have been made for shaping the light with various desired structures and properties [32–41], and these studies have fueled the research interest in beam synthesis and engineering as one of the interdisciplinary areas beyond optics and photonics.

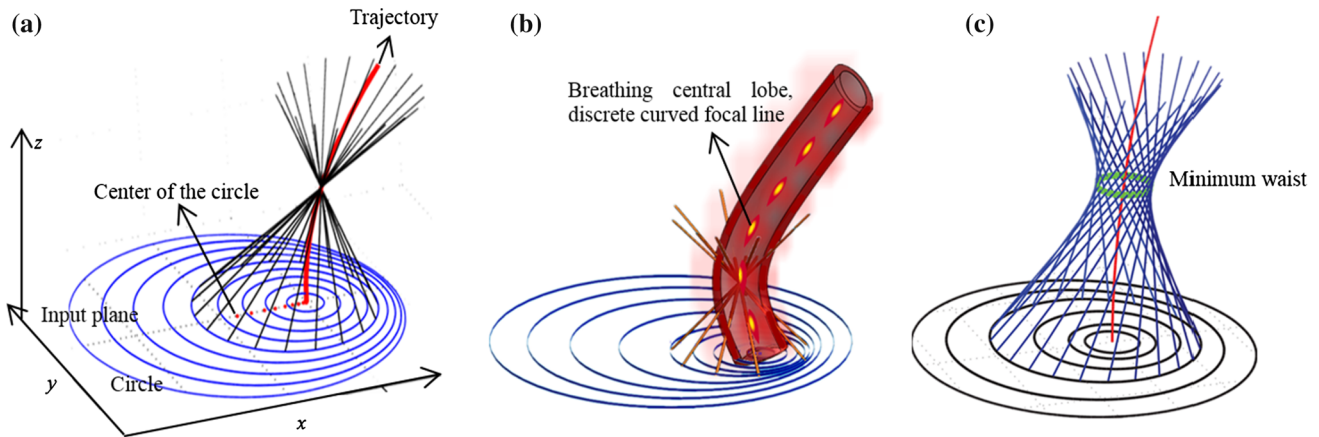
In this paper, we provide a brief overview of our recent work on spatially shaped accelerating beams along arbitrary trajectories, including the self-accelerating Bessel-like beams (self-bending in transverse direction) with or without vorticity, self-breathing Bessel-like beams (with self-pulsating peak intensity along propagation direction), as well as nonparaxial (with large bending angles) Bessel-like beams [37]. In addition, we propose and demonstrate a new class of self-accelerating beams that can also undergo self-propelling (with multiple rotating intensity blades) during propagation. Based on the phase modulation and superposition method, the ability of designing various kinds of accelerating beams with arbitrary trajectories and symmetric transverse profile is illustrated. These spatially shaped dynamical beams are gifted with properties such as resistance to diffraction, capability of self-healing, controllable beam profiles and tunable trajectories, which make them particularly attractive for many applications.

## 2 Paraxial accelerating beams

Under the paraxial approximation, the propagation of an optical beam obeys the Fresnel diffraction integral:

$$u(X, Y, Z) = \frac{1}{2\pi i Z} \iint u(x, y, 0) e^{i\frac{(X-x)^2 + (Y-y)^2}{2Z}} dx dy, \quad (1)$$

where  $u(x, y, 0) = \exp(-(x^2 + y^2)/w^2) \exp(iQ(x, y))$  is a phase-modulated input optical field with the transverse ( $X, Y; x, y$ ) and longitudinal ( $Z$ ) coordinates being scaled from real coordinate ( $x', y', z'$ ) by  $\alpha$  and  $k\alpha^2$ , respectively. Here,  $w$  is the beam width,  $k$  is the wave number and  $\alpha$  is an arbitrary length scale. The input phase pattern  $Q(x, y)$  determines the ray trajectories in free space. These rays can be designed to create a focal curve ( $f(Z), g(Z), Z$ ) and Bessel function profile, namely Bessel-like beam, as shown in Fig. 1. Specifically, any point on this curve is the apex of a conical ray bundle emanating from a circle on the input plane. The radius and the location of the center of this circle are determined by the formulas derived in Refs. [28, 29]. The center in particular is the point at which the tangent of the trajectory at ( $f(Z), g(Z), Z$ ) intersects the input plane. This scenario is schematically plotted in Fig. 1a. In this context, if the input condition is obstructed along some of these circles, the main lobe at the corresponding distance will disappear. By removing input annuli (groups of these expanding circles) in a periodic fashion, energy periodically disappeared from the curved trajectory which exhibits a pulsating and breathing central lobe and a discrete curved focal line, namely breathing self-accelerating Bessel-like beam, as shown in Fig. 1b [32]. On the other hand, the rays from a circle at the input plane can also be given angular momentum to create hyperboloids (Fig. 1c) with a minimum waist (Fig. 1c) that guides along a



**Fig. 1** (Color online) Schematic of the principle: Rays emitted from expanding circles on the input plane intersect on the specified focal curve. **a** A self-accelerating Bessel-like beam, where the dots are the shifting circle centers [28], **b** A breathing self-accelerating Bessel-like beam, where some circles are removed alternately so that the main lobe experiences breathing during propagation [32], **c** A self-accelerating vortex Bessel-like beam, where rays are skewed from each other rather than converging as in (a) and (b) [30]

predefined trajectory. In this way, vortex accelerating Bessel-like beams are produced [30, 31].

In the following sections, we will show that the transverse beam profiles generated via these schemes can be described approximated by the zero-order Bessel function  $J_0(\beta r)$  (Fig. 1a, b) and higher-order Bessel function  $J_m(\beta r)$  in polar coordinates ( $\beta$  is the normalized transverse wave number of the Bessel beam,  $r = (\delta X^2 + \delta Y^2)^{1/2}$ , and  $m$  is the order of vorticity in Fig. 1c), and they “focus” at controllable distances while keeping the beam structure remarkably invariant.

### 2.1 Self-accelerating Bessel-like beams along smooth trajectories

Let us start with some calculations for designing the accelerating Bessel-like beams, which are named so because the beam can bend along curved trajectories with a Bessel-like transverse beam profile. Note that the first partial derivatives of the phase  $Q$  satisfy  $Q_x = (f - x)/Z$ ,  $Q_y = (g - y)/Z$ , where  $(x, y)$  marks the starting point of any ray in that conical ray bundle, as shown in Fig. 1a. The continuum of points  $(x, y, 0)$  creates a geometric circle  $C(Z)$  with center  $(x_0, y_0)$  and radius  $R(Z)$  on  $Z = 0$ , which can be viewed as the isocurve of a function  $Z(x, y)$ . Since  $Q$  should be twice continuously differentiable, its mixed partials must be equal  $Q_{xy} = Q_{yx}$ , yielding  $(x - x_0(Z))Z_y = (y - y_0(Z))Z_x$ , where  $x_0(Z) = f(Z) - Zf'(Z)$ ,  $y_0(Z) = g(Z) - Zg'(Z)$ . After some long algebraic procedure, the phase  $Q$  is shown to follow the formulas:

$$Q(x, y) = \frac{1}{2} \int_0^Z \left\{ [f'(\zeta)]^2 + [g'(\zeta)]^2 - 1 \right\} d\zeta - \frac{(f - x)^2 + (g - y)^2}{2Z}, \tag{2}$$

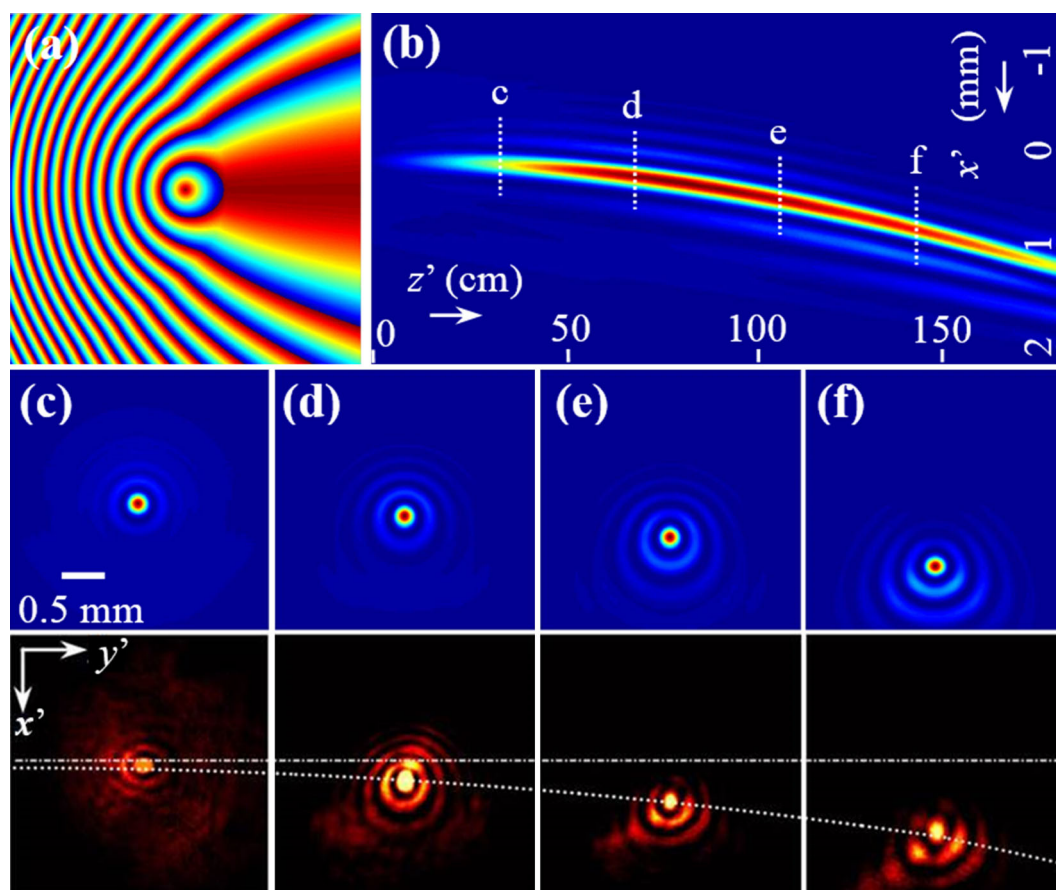
$$Z^2 = [x - f(Z) + Zf'(Z)]^2 + [y - g(Z) + Zg'(Z)]^2. \tag{3}$$

The above algorithm is well defined only when Eq. (3) has a unique solution for  $Z$ , which is equivalent to the following inequality:

$$R'(Z) > Z \left[ (f''(Z))^2 + (g''(Z))^2 \right]^{1/2}. \tag{4}$$

Equation (4) defines the maximum propagation distance  $Z_{\max}$  of the focal curve. Beyond  $Z_{\max}$ , a straight trajectory along its tangent direction is defined, so that the beam keeps a nondiffracting profile of a tilted standard Bessel beam. These Bessel-like beams inherit from standard ones the properties of diffractionless propagation and self-healing in addition to propagating along arbitrary trajectories. As examples, we have demonstrated parabolic, snake-like, hyperbolic as well as 3D spiraling trajectories [28, 29].

Figure 2 presents an example of the Bessel-like beam following the parabolic trajectory  $((f(Z), g(z)) = (Z^2/40, 0))$ . The phase structure shown in Fig. 2a is the one used to modulate a broad Gaussian input beam. We have numerically simulated the evolution of the wave using a split-step Fourier algorithm. The beam evolution recorded in the  $X$ - $Z$  plane shows that the main peak follows a parabola up to 140 cm and a straight line thereafter (Fig. 2b). During acceleration, the transverse profiles (Fig. 2c–f) are recorded, which reveal a remarkably persistent main lobe (it is a very good fit to the function  $J_0(r)$ , although elongated along the axis of acceleration). Beyond  $Z_{\max}$ , acceleration stops and the symmetric Bessel profile is restored as our wave is essentially a tilted standard Bessel beam. And Eq. (1) can be rewritten as



**Fig. 2** (Color online) Numerical and experimental demonstrations of a self-accelerating Bessel-like beam along a parabolic trajectory. **a** Modulated input phase for the Bessel-like beam, **b** numerically simulated side-view propagation of the generated accelerating beam, **c–f** snapshots of numerical (middle row) and experimental (bottom row) transverse intensity patterns taken at different planes marked by the dashed lines in **(b)** [28, 29]

$$u(X, Y, Z) = J_0 \left( \sqrt{(X - \eta Z)^2 + Y^2} \right) \times \exp(i\eta X - i(1 + \eta^2)Z/2), \quad (5)$$

where  $J_0$  is the Bessel function and  $\eta$  is a constant. Therefore, at any  $Z$  plane, the optical field around the focus behaves like a Bessel function modulated by a plane wave.

To experimentally realize such self-accelerating Bessel-like beams, we utilize computer-generated holography via a spatial light modulator (SLM) that is used to modulate the phase distribution of the input light field. A Gaussian beam emitted from an argon ion laser passes through the SLM programmed by a computer-generated hologram obtained by calculating the interference between the initial optical field and a tilted plane wave. Upon reflection from the hologram, the encoded beam information is reconstructed via a typical  $4f$  system with spatial filtering. By shifting the CCD camera along  $z$  direction, we detect the images and record the movie at different propagation distances [29]. Figure 2c–f shows that the experimental results (bottom

row) are in good agreement with numerical simulations (middle row). Using a similar approach, we have designed and demonstrated Bessel-like beams that propagate along several different types of trajectories, including sinusoidal, hyperbolic, and hyperbolic secant trajectories lying in a plane (2D trajectories) as well as spiraling trajectories in 3D space. In all cases, the observed trajectories agree well with the theoretical predictions. In addition, self-healing has also been demonstrated. In the latter work, the beam was partially blocked by an opaque wire thus losing its main lobe. During the subsequent propagation, the beam recovered its main lobe and restored its structure as expected [28, 29].

## 2.2 Breathing self-accelerating Bessel-like beams

As mentioned in the previous sections, a self-accelerating Bessel-like beam can be designed to exhibit a discrete (or breathing) curved trajectory by employing the scheme shown in Fig. 1b [32]. In this way, the intensity of the



central main lobe of these self-accelerating Bessel-like beams displays a pulsating or breathing feature, thus named “breathing self-accelerating Bessel-like beam”. More specifically, the term “breathing” refers to that the central lobe of the beam disappears and reappears periodically as the energy of the beam alternately switches between the main lobe and the outer rings during propagation. These beams are produced by employing the initial condition  $u(x, y, 0) = \exp(- (x^2 + y^2)/w^2)\exp(iQ_1(x, y))$ , where the phase  $Q_1$  comes from a binarization of  $Q$  (the continuous phase of the Bessel-like beams), for instance:

$$Q_1 = \begin{cases} 0, & \min\_Q \leq Q < \min\_Q + c, \\ 1, & \min\_Q + c \leq Q < \min\_Q + 2c, \\ 0, & \min\_Q + 2c \leq Q < \min\_Q + 3c, \\ 1, & \min\_Q + 3c \leq Q < \min\_Q + 4c, \\ \vdots & \vdots \\ \begin{cases} 0, & 2[n/2] = n, \\ 1, & 2[n/2] \neq n, \end{cases} & \min\_Q + nc \leq Q \leq \max\_Q, \end{cases} \quad (6)$$

where  $\max\_Q$  and  $\min\_Q$  are the maximum and minimum values of  $Q$ , respectively,  $c$  is a constant satisfying the requirement  $0 < c < (\max\_Q - \min\_Q)/2$ , the parameter  $n = [\max\_Q/c]$  is the maximum integer of  $\max\_Q/c$  and the square brackets are for integer conversion. After being modulated by the phase  $Q_1$ , the beam carrying a Bessel-like profile exhibits pulsation during propagation, as the peak intensity oscillates between the main lobe and the outer rings periodically.

Specifically, we obtain  $Q_1$  from the following rules:

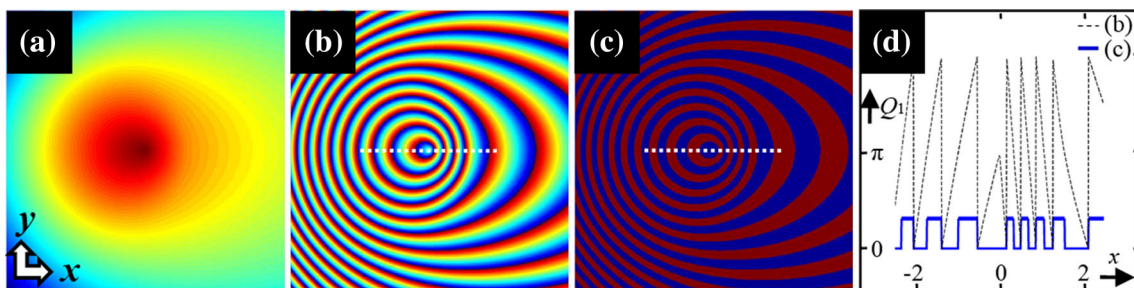
$$Q' = Q - 2c \left\lfloor \frac{Q}{2c} \right\rfloor, \\ Q_1 = \begin{cases} 1, & Q' \geq c, \\ 0, & Q' < c. \end{cases} \quad (7)$$

Figure 3 presents a typical example of the process used to produce  $Q_1$ . We start from a phase  $Q$  shown in Fig. 3a,

which smoothly decreases along the radial direction. Then,  $Q$  is divided into a series of “periodic” rings  $Q'$  (Fig. 3b), which is the remainder of the  $Q$  when dividing by  $2c$  ( $c = \pi$ ). Here, the “period”  $2c$  just means the modulated width  $\Delta Q$  for each ring. Finally, a “well”-shaped phase  $Q_1$  (Fig. 3c) is obtained from  $Q'$  by using Eq. (7). One can directly compare the phase structure between  $Q'$  and  $Q_1$  in Fig. 3d, where the phase along the dotted line shown in Fig. 3b, c is plotted. With this phase  $Q_1$ , the beams created in  $Z > 0$  possess breathing transverse Bessel-like field patterns along a desired trajectory ( $f(Z), g(Z), Z$ ) in free space.

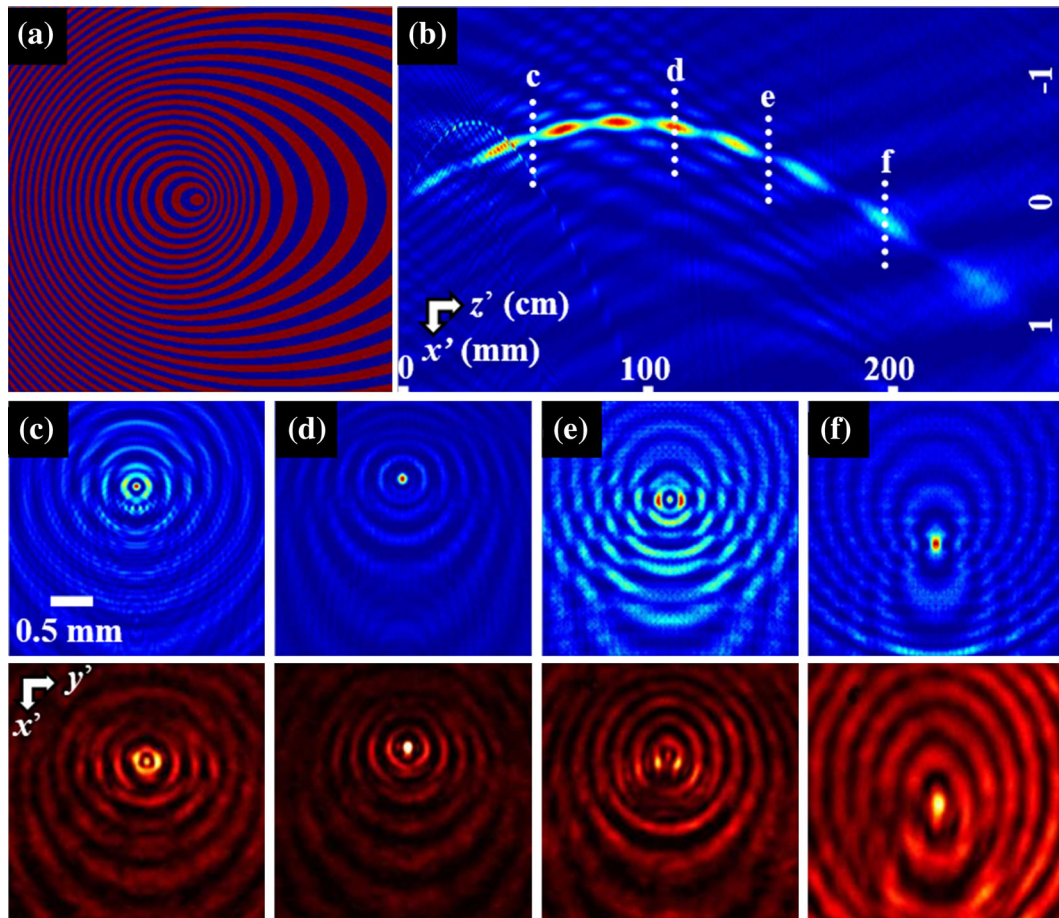
The intuitive picture behind the evolution of the breathing beams is illustrated in Fig. 1b. Due to the “periodically” modulated phase  $Q_1$ , the interval of the breathing trajectory is determined by the annular width  $\Delta Q$  at the input plane. The intensity peak alternately switches its location between the main lobe and the outer rings of the Bessel-like beam. As a typical example, we consider a beam propagating along a smooth hyperbolic trajectory in an oscillating configuration with a binary phase as shown in Fig. 4a. Figure 4b shows the numerical results of the breathing beam propagation. One can see that there are nine quasi-periodic breathings in the main lobe of the beam for the propagation distance of 200 cm. Indeed, the quasi-period of the breath can be controlled with ease by adjusting the modulation width  $c$  of the rings in the phase function  $Q_1$ . Transverse intensity patterns taken at different  $Z$  are shown in Fig. 4c–f, where it clearly illustrates that the beam possesses the Bessel-like intensity profile, and the location of peak intensity alternates between the central lobe and outer rings of the Bessel-like beam during propagation. It can be verified that these breathing Bessel-like beams also preserve their self-healing property.

Our detailed analysis shows that the pulsating feature of the peak intensity can also be introduced to the Bessel-like beams following other trajectories such as parabolic, hyperbolic secant, and 3D trajectories. These beams may be



**Fig. 3** (Color online) Phase structure for the generation of breathing accelerating beams as obtained from theoretical analysis. **a** The original phase  $Q$  obtained from Eq. (2), **b**, **c** the phase  $Q'$  and  $Q_1$  obtained from Eq. (7), **d** plots of the phase  $Q_1$  and  $Q'$  along the dotted lines in (b) and (c) [32]





**Fig. 4** (Color online) Numerical and experimental demonstrations of a breathing self-accelerating beam along a hyperbolic trajectory. **a** Binary-modulated input phase for the breathing self-accelerating beam with  $f(Z) = \sqrt{0.64Z^2 - 32Z + 800} - \sqrt{800}$ ,  $g(Z) = 0$ , **b** numerically simulated side-view propagation of the generated beam, **c–f** numerical (middle row) and experimental (bottom row) snapshots of the transverse intensity patterns taken at different transverse planes marked by the dotted vertical lines in **(b)** [32]

particularly attractive for various applications including particle trapping and micromanipulation.

### 2.3 Vortex self-accelerating beams

Optical vortex beams, namely the beams that carry orbital angular momentum (OAM), have been an interesting subject of research for many decades [42–45]. These beams have a helical phase structure described as  $\exp(im\theta)$ , and  $m$  indicates the multiple of  $2\pi$  round-trip phase around the vortex singularity, which is called the topological charge. Although attempts have been made to combine the properties of accelerating Airy beams and vortex beams, it remains a challenge to impose the OAM even onto the main lobe of a simple asymmetric 2D Airy beam [46]. Recently, we have proposed and demonstrated a new kind of self-accelerating beams carrying OAM in the form of accelerating vortex Bessel-like beams [30, 31]. These beams propagate along a desired trajectory  $(f(Z), g(Z), Z)$ , having a

profile that resembles a high-order Bessel function  $J_m$  with an invariant central dark core.

An accelerating vortex beam can be constructed using an approach similar to that developed in Ref. [28]. The initial beam profile is written in terms of a smooth amplitude function modulated by a helical phase  $Q_2$  as  $u(x, y, 0) = A(x, y)\exp(iQ_2(x, y))$ . Utilizing Eq. (1), we get

$$u(X, Y, Z) = \frac{1}{2\pi i Z} \iint A(x, y) \times \exp \left[ iQ_2(x, y) + i \frac{(X-x)^2 + (Y-y)^2}{2Z} \right] dx dy \quad (8)$$

From this equation, the overall phase of the vortex beam is

$$\psi(X, Y) = Q_2(x, y) + \frac{(X-x)^2 + (Y-y)^2}{2Z}. \quad (9)$$

By applying the stationary phase method to the Fresnel integral, we obtain

$$\psi_x = \frac{X - x}{Z}, \quad \psi_y = \frac{Y - y}{Z}. \quad (10)$$

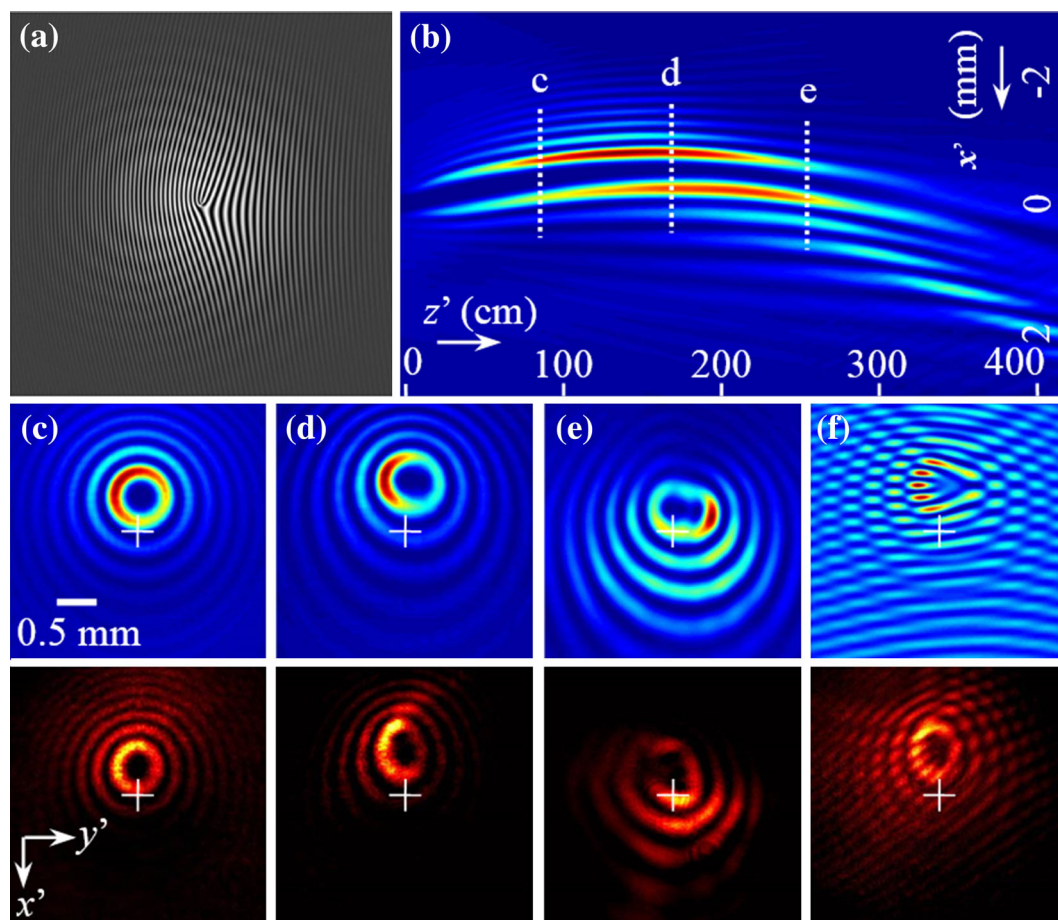
Via the modulation of the phase  $\psi$ , the rays originating from a circle at the input plane lead to a vortex caustic at distance  $Z$ , which creates an oblique conical-like surface with a nonzero minimum waist (Fig. 1c).

As a typical example, we show in Fig. 5 the vortex beam with a topological charge  $m = 3$  propagating along a hyperbolic trajectory. Following a similar algorithm to obtain the Bessel-like accelerating beams, the phase  $\psi$  can be solved as shown in Fig. 5a. The recorded trajectory of the dark core is in very good agreement with the theoretical prediction (Fig. 5b). The beam propagates along a hyperbolic curve and reaches a maximum intensity and the ultimate deviation at about 180 cm. The beam pattern looks similar to the higher-order Bessel beam  $J_3$  profile with a central diffraction-free dark core (Fig. 5c–e), while the measured beam sizes at different propagation distances indicate that the main ring maintains almost a constant

width during propagation. In addition, the topological charge remains unchanged even though the beam propagates along the curved trajectory. Such a dynamical optical beam combining features from the Bessel, Airy and vortex beams may provide a new tool for optical trapping and manipulation of microparticles when combined with optical tweezers technique [31].

### 2.4 Propelling self-accelerating beams

Optical propelling beams, namely the dynamical beams that exhibit multiple rotating intensity blades, have attracted great attention both due to their fundamental interest and due to their potential applications [47–51]. So far, a number of techniques have been proposed to create a rotating optical field intensity, including revolving mirrors [48, 52], interference of Laguerre-Gaussian modes [49, 50], rotating apertures [51], and the Moiré-pattern technique [53, 54]. Compared with previously developed methods,



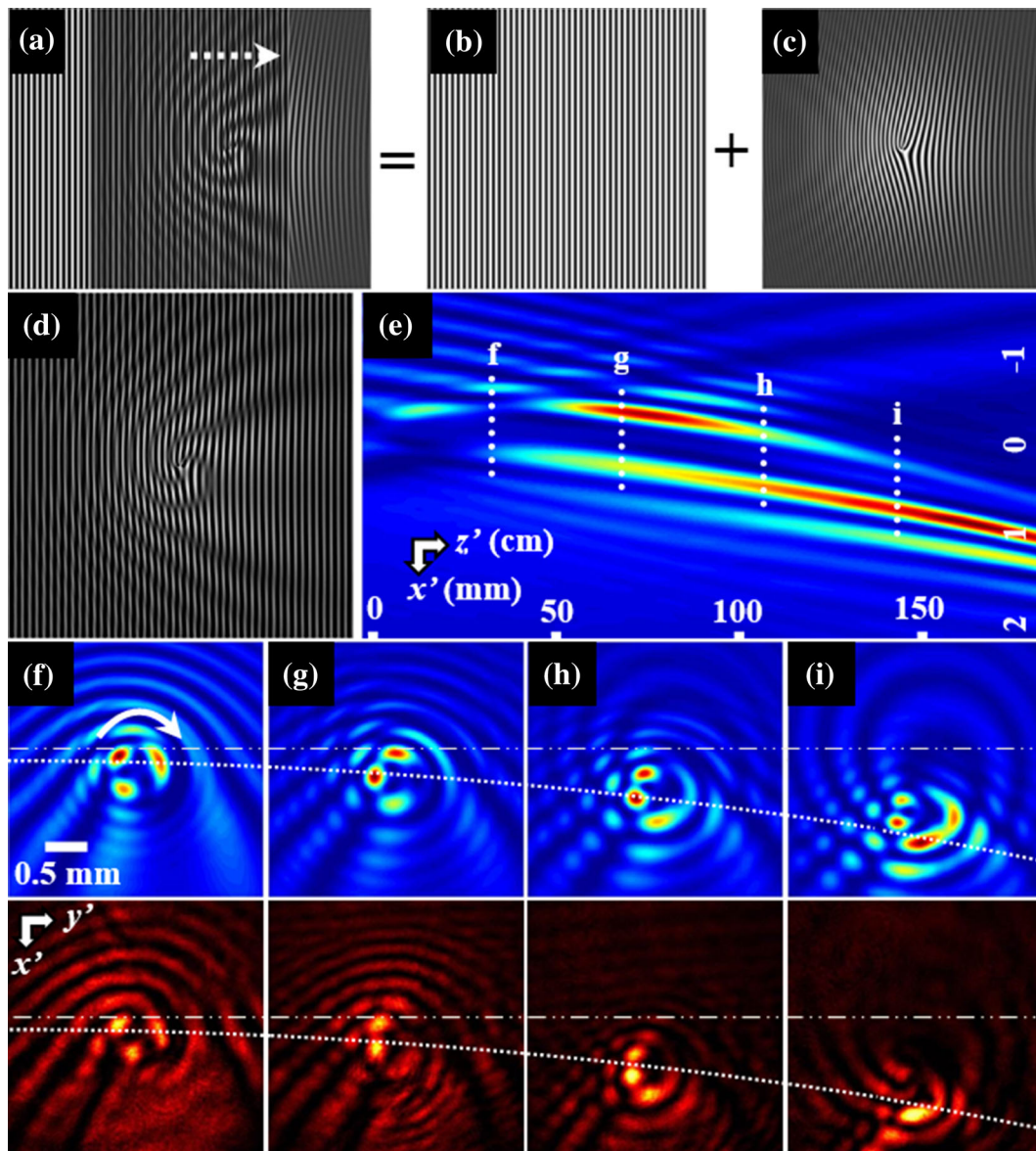
**Fig. 5** (Color online) Numerical and experimental demonstrations of a triply charged vortex beam traveling along a hyperbolic trajectory. **a** Computer-generated hologram with desired phase information, **b** numerically simulated side-view propagation of the generated beam, **c–e** numerical (middle row) and experimental (bottom row) transverse beam patterns taken at different marked positions in **(b)**, **f** interferograms from the pattern **(d)** with an inclined plane wave, showing the triply charged vorticity



the Moiré technique we developed recently has the advantage that all the rotation features of the beams can be changed at ease without any mechanical movement or phase-sensitive interference. It is also of great interest to introduce the propelling intensity into accelerating beams based on the Moiré technique. In this section, such propelling accelerating beams are created and analyzed theoretically and demonstrated experimentally. In theory, these beams are generated by overlapping a vortex accelerating beam and a plane wave, i.e.,  $u(x, y, 0) = u_1(x, y, 0) + u_2(x, y, 0)$ , where  $u_1(x, y, 0) = \exp(-(x^2 + y^2)/w^2)$

$\exp(iQ_2(x, y))$  is a vortex accelerating beam and  $u_2(x, y, 0) = \exp(ikx)$  is a plane wave.

To create the desired Moiré fringes, we overlap a moving straight-line grating (computer-generated hologram from two plane waves) and a fork-type grating with topological charge of  $m = 3$  (the interference between a plane wave and a self-accelerating vortex beam), as shown in Fig. 6a–c. Based on the superposed gratings (Fig. 6d), Fig. 6e clearly illustrates that the generated beam propagates along a parabolic trajectory. Figure 6f–i shows numerically and experimentally retrieved transverse intensity



**Fig. 6** (Color online) **a–d** The Moiré pattern used for generating a multiblade propelling accelerating beam formed by overlapping/translating a straight-line grating (**b**) with respect to a fork-type grating (**c**), where the fork-type grating is associated with a triply charged vortex, **e** numerically simulated side-view propagation of the propelling beam, **f–i** numerical (third row) and experimental (bottom row) transverse intensity patterns at different longitudinal positions marked in (**e**); the dashed arrow in (**a**) indicates the moving direction of the straight-line grating, and the solid arrow in (**f**) indicates the rotating direction of the dynamical beam pattern

patterns from the Moiré fringes of Fig. 6e at different longitudinal positions. From these figures, it is obvious that the beam profile consists of three intensity blades, with each blade rotating around the central dark core due to moving of the straight-line grating. In this scheme, the number of intensity blades is determined by the topological charge of the vortex. The rotation direction (marked by the white solid arrow) depends on the direction of the grating movement and the sign of the topological vortex charge, and the rotation speed is proportional to the speed of the grating motion. Overall, the Moiré pattern technique allows convenient changes of the number of intensity blades, as well as the rotation direction and speed of the intensity pattern. The generation of these propelling beams represents a transition from linear translation to rotation and from the vortex phase singularity to azimuthal intensity variation without using any mechanical rotating system.

Likewise, a propelling and accelerating beam can also propagate along different trajectories, as demonstrated theoretically and experimentally. Figure 7 illustrates typical experimental results of the generated propelling beams along a 3D trajectory ( $f(Z) = 5 \tanh [0.12(Z - 10)] + 5$ ,  $g(Z) = 6 \operatorname{sech} [0.12(Z - 10)]$ ). Figure 7a shows schematically that the reconstructed propelling beam from the Moiré technique propagates along the 3D trajectory. Figure 7b–d

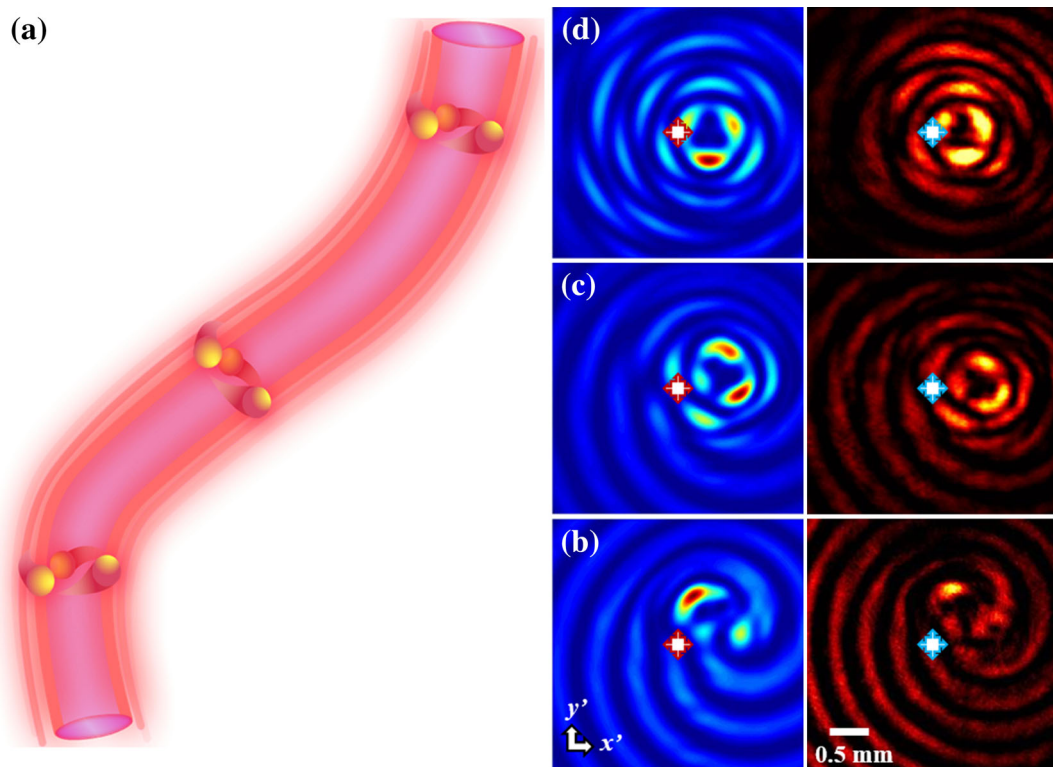
show numerical and experimental results of dynamical transverse intensity patterns obtained with this technique at different longitudinal positions. As seen from these results, the rotating beam profile is primarily composed of three rotating blades. These propelling beams bring about the possibility for dynamic microparticle manipulation based on the Moiré technique which is inherently immune to environmental perturbations. These fine-shaped dynamical light beams may also find applications in micromachining and in developing multifunctional rotating optical tweezers for biological research.

### 3 Nonparaxial self-accelerating Bessel-like beams

In this section, we discuss briefly Bessel-like beams beyond the paraxial approximation [37]. Nonparaxiality plays an important role when the transverse oscillations of a beam are in a scale comparable to or smaller than the operating wavelength. Under this condition, the wave evolution is governed by the vector Helmholtz equation:

$$(\nabla^2 + k^2)\mathbf{E} = \mathbf{0}. \tag{11}$$

According to which, the wave vector components satisfy the dispersion relation  $k_z = (k^2 - k_x^2 - k_y^2)^{1/2}$ . For a given input condition, the Helmholtz equation is solved by the



**Fig. 7** (Color online) The three-blade propelling accelerating beam propagates along a 3D trajectory. **a** Schematic of the beam propagation, **b–d** numerical (middle column) and experimental (right column) transverse intensity patterns taken at different propagation distances as illustrated in **(a)**



Rayleigh–Sommerfeld formula for one component of the electric field vector potential

$$u(\mathbf{R}) = -\frac{k^3 Z}{2\pi} \iint u(\mathbf{r}) G(k|\mathbf{R} - \mathbf{r}|) e^{ik|\mathbf{R} - \mathbf{r}|} d\mathbf{r}, \quad (12)$$

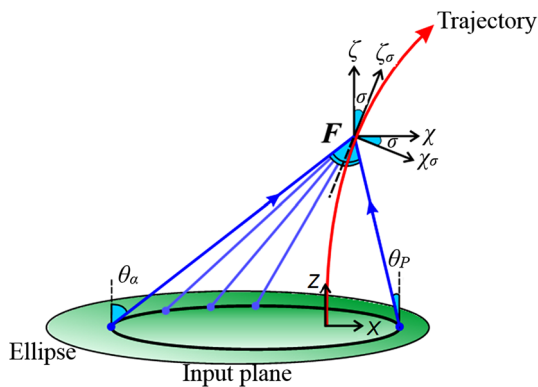
where we assume the input to be formulated by a slowly varying envelope times a phase factor, i.e.,  $u(\mathbf{r}) = U(\mathbf{r})e^{iQ_3(\mathbf{r})}$ ,  $G(r) = ir^{-2} - r^{-3}$  is the Green's function,  $\mathbf{R} = (X, Y, Z)$  is the observation position, and  $\mathbf{r} = (x, y, 0)$  spans the input plane.

By modulating the input light field with the phase  $Q_3$ , the rays create a continuous focal line in the free space with a given parametric trajectory  $\mathbf{F}(Z) = (f(Z), g(Z), Z)$ . At any point on this line, rays emanating from a geometric locus in the input plane intersect and interfere to create a Bessel-like profile. Therefore, one can realize a Bessel-like beam following the pre-designed trajectories through employing the phase modulation shown below at the input plane

$$(Q_{3x}, Q_{3y}) = \frac{k}{|\mathbf{F}(Z) - \mathbf{r}|} (f(Z) - x, g(Z) - y). \quad (13)$$

For the trajectory  $(f(Z), 0)$ , here  $g(Z) = 0$ , Fig. 8 shows the ray-optics schematic for the diffraction of the potential function  $u(\mathbf{r})$ . Rays emitted from expanding ellipses interfere to create a curved focal line (the curve trajectory).

Using the concept of auxiliary vector potentials, the electromagnetic problem of a vectorial Bessel-like beam in the nonparaxial region can be reduced to the dynamics of a scalar wave function (a single component of a vector potential) according to the above diffraction integral. Again, the phase  $Q_3$  satisfies  $Q_{3xy} = Q_{3yx}$ , from which we can obtain



**Fig. 8** (Color online) Ray-optics schematic for the diffraction of the potential function  $u(\mathbf{r})$ . Shown on the  $Y = 0$  plane are the global coordinates  $X, Z$ , and at the focal point the local coordinates  $\chi, \zeta$  [37]

$$Z_x y \left( x - f - \frac{Z}{f'} \right) = -Z_y \left[ y^2 + \frac{Z}{f'} (x - f) + Z^2 \right]. \quad (14)$$

The shape of the ellipse satisfies the following equation:

$$\frac{(x - x_0)^2}{a^2} + \frac{y^2}{b^2} = 1, \quad (15)$$

where  $a(Z) = Z[(\omega - 1)(1 + \omega f'^2)]^{1/2}$ ,  $b(Z) = Z(\omega - 1)^{1/2}$ ,  $x_0(Z) = f - \omega Z f'$ , and  $\omega(Z)$  is an arbitrary dimensionless function.

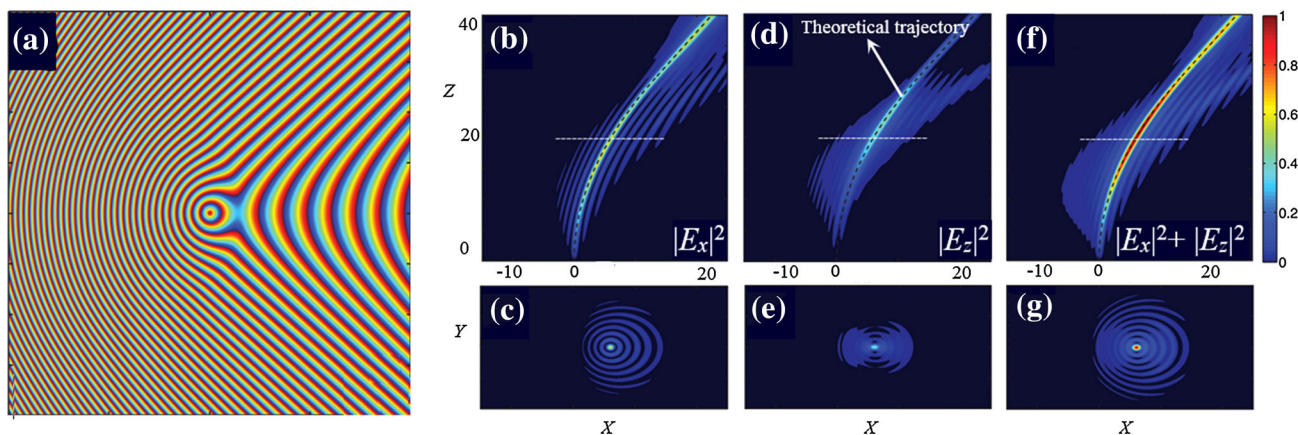
After some analyses, the phase at any point  $\mathbf{r}$  on input plane is given by  $Q_3(\mathbf{r}) = P(Z) - k|\mathbf{R} - \mathbf{r}|$ , where  $Z$  corresponds to the elliptical locus  $C(Z)$  passing from this point,  $P(Z) = k \int_0^Z \frac{\cos[\gamma(z)]}{\cos[\sigma(z)]} dz$ ,  $dz/\cos \sigma = dz(1 + f'^2)^{1/2}$ , and

the cone half angle  $\gamma = \arctan\left(\frac{\omega - 1}{1 + \omega f'^2}\right)^{1/2}$ , which describes the width of the beam's main lobe. Providing that the input amplitude satisfies  $U = (2\pi k_\perp \rho_\sigma)^{-1/2} e^{i\pi/4}$  (where  $k_\perp = k \sin \gamma$  and  $\rho_\sigma$  is the distance from the axis of the beam), the beam evolution can be expressed as

$$u(\mathbf{F} + \delta\mathbf{R}_\sigma) \approx J_0(k_\perp \rho_\sigma) \exp \left[ ik_\parallel \left( \frac{Z}{\cos \sigma} + \zeta_\sigma \right) \right], \quad (16)$$

where  $k_\parallel = k \cos \gamma$ . Equation (16) clearly represents an ideal diffraction-free Bessel beam that propagates with wave vector  $k_\parallel$ . Therefore, the ray-optics approximation is consistent with the fact that the Helmholtz equation supports the Bessel beam solutions.

As an example, the Bessel-like beam is designed to propagate again along a parabolic trajectory. In the simulations shown in Fig. 9, we adopt the trajectory function  $(f(Z), g(Z)) = (0.015Z^2, 0)$  and parameter  $\gamma = 30^\circ$ ; here, all distances are measured in wavelengths. For a standard nonaccelerating Bessel beam, the corresponding subwavelength FWHM of the central lobe can be obtained with  $\gamma$  value. By employing the phase structure presented in Fig. 9a, a fully vectorial nonparaxial accelerating Bessel-like wave is obtained whose intensity follows a parabolic trajectory (Fig. 9b–g). Note that the trajectory of the simulated beam agrees well with the pre-designed one. In Fig. 9e, the longitudinal component  $E_Z$  is initially weak, but its magnitude increases to become comparable to the transverse component  $E_X$  as the trajectory of the beam gets more inclined. This is because the more the wave loses its transverse nature, the more it departs from the paraxial regime. For the total electric energy density (Fig. 9g), one sees that the overall beam profile inherits the expected Bessel-like form with familiar ring structure (elongated along the direction of acceleration). And the presence of the longitudinal component does not affect the Bessel profile significantly.



**Fig. 9** (Color online) Simulation results for a nonparaxial accelerating Bessel-like beam with a parabolic trajectory. **a** Input phase structure, **b** evolution of  $|E_x|^2$  on the plane  $Y = 0$ , and **c** its transverse profile on the plane  $Z = 20$  as marked in **(b)** with a white dashed line, **d–e** the same for  $|E_z|^2$ . **f–g** The same for the electric energy density  $|E_x|^2 + |E_z|^2$ . **b, d, f** The black dashed curves in the beam center lobe indicate the theoretical trajectory. The maps have the same color code to allow direct comparison [37]

In brief, the fully vectorial nonparaxial accelerating Bessel-like waves have been demonstrated by an appropriate modification of the conical ray pattern of the standard Bessel beams. We have found that these numerical results are in good agreement with theoretical design, in cases of parabolic, hyperbolic, hyperbolic secant, and 3D trajectories. In contrast to caustic-based beams which are highly asymmetric in their transverse intensity distribution, these new accelerating beams provide the convenience of a symmetric Bessel-like intensity profile which is much needed for many applications.

#### 4 Potential applications

All of the beams discussed above, which possess nearly symmetric transverse beam profiles together with features of nondiffracting, self-healing, and tunable trajectories, may be particularly attractive for various applications already proposed [7–16], including optical microparticles manipulation, generation of special light bullets, generation of curved plasma channels, routing of surface plasmon polaritons, generation of self-bending electron beams without any external field, and high-resolution imaging and microscopy. These promising applications have further advanced the research of self-accelerating beams and beam shaping in general in the fields beyond optics and photonics.

Optical manipulation of particles with lasers is an indispensable tool across many branches of science, including molecular biology, medicine, nanotechnology, atmospheric science and colloidal physics. In this field, the leading tools are optical tweezers with specially shaped beam profiles, which apply gradient forces and radiation pressure on trapped transparent particles. With Bessel-like profile, these beams can trap the micro-objects into the central area of the beams and transport them to the desired

destination along predefined curved route, promising for long-range material transport. As an “optical wrench”, the self-propelling beams are suitable for particle grouping and rotation owing to their dynamical intensity blades. Meanwhile, rotation of trapped particles offers another important degree of freedom for optical manipulation, promising for applications in biotechnology.

In atmospheric sciences, these beams possessing high transmission efficiency and self-healing properties are desirable for light propagation through the atmospheric turbulence or other complex scattering media. Meanwhile, with the self-accelerating property, these beams can be used to generate curved plasma channels and even to control electric discharges.

#### 5 Summary

In this paper, we have reviewed briefly the conception and development of dynamical spatially shaped Bessel-like accelerating beams with a central symmetric transverse intensity profile traveling along arbitrary trajectories. Based on our design, the self-accelerating Bessel-like beams, breathing Bessel-like beams and vortex Bessel-like beams have all been demonstrated. In addition, we have shown that it is feasible to create self-propelling while simultaneously self-accelerating optical beams by the use of the phase design combined with the Moiré technique. Finally, such specially designed accelerating beams might be synthesized beyond the paraxial region based on the ray-optics interpretation of the rigorous Rayleigh–Sommerfeld diffraction formula. Our results may lead to new possibilities for optical beam shaping and beam engineering that may find a variety of potential applications, while continued research on accelerating beams keeps bringing up new

momentum to the arena of optical beam shaping and applications [55–64].

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